NAME _____ DATE _____

11–5 Square Roots of Variable Expressions

Objective: To find square roots of variable expressions and to use them to solve equations and problems.

Property

> **Property of Square Roots of Equal Numbers** For any real numbers r and s:
> $r^2 = s^2$ if and only if $r = s$ or $r = -s$.

CAUTION When you are finding the principal square root of a variable expression, you must be careful to use absolute value signs when needed to ensure that your answer is positive. For example, $\sqrt{x^2} = |x|$, not x.

Example 1   Simplify: **a.** $\sqrt{144x^2}$ **b.** $\sqrt{25n^8}$ **c.** $\sqrt{12a^3}$

Solution   **a.** $\sqrt{144x^2} = \sqrt{144} \cdot \sqrt{x^2}$
$= 12|x|$

b. $\sqrt{25n^8} = \sqrt{25} \cdot \sqrt{n^8}$
$= \sqrt{25} \cdot \sqrt{(n^4)^2}$
$= 5n^4$ (n^4 is always nonnegative)

c. $\sqrt{12a^3} = \sqrt{4 \cdot 3 \cdot a^2 \cdot a}$
$= \sqrt{4} \cdot \sqrt{a^2} \cdot \sqrt{3} \cdot \sqrt{a}$
$= 2|a|\sqrt{3a}$

Simplify.

1. $\sqrt{81x^2}$ **$9|x|$**

2. $\sqrt{121x^2}$ **$11|x|$**

3. $\sqrt{20x^2}$ **$2\sqrt{5}|x|$**

4. $\sqrt{45x^4}$ **$3\sqrt{5}x^2$**

5. $-\sqrt{25x^2}$ **$-5|x|$**

6. $-\sqrt{16c^4}$ **$-4c^2$**

7. $-\sqrt{64d^8}$ **$-8d^4$**

8. $-\sqrt{98n^6}$ **$-7\sqrt{2}|n^3|$**

9. $\sqrt{225y^4}$ **$15y^2$**

10. $\sqrt{400a^6b^4}$ **$20|a^3|b^2$**

11. $\sqrt{81m^{12}}$ **$9m^6$**

12. $\sqrt{441n^6}$ **$21|n^3|$**

13. $\pm\sqrt{75x^2y^3}$ **$\pm 5|xy|\sqrt{3y}$**

14. $\pm\sqrt{60x^6y^4}$ **$\pm 2|x^3|y^2\sqrt{15}$**

15. $-\sqrt{121x^2y^2}$ **$-11|xy|$**

16. $-\sqrt{900a^4b^6}$ **$-30a^2|b^3|$**

17. $\pm\sqrt{\dfrac{81x^8}{100}}$ **$\pm\dfrac{9}{10}x^4$**

18. $\pm\sqrt{\dfrac{121}{225x^{10}}}$ **$\pm\dfrac{11}{15|x^5|}$**

19. $\sqrt{\dfrac{x^4y^8}{9z^2}}$ **$\dfrac{x^2y^4}{3|z|}$**

20. $\sqrt{\dfrac{32m^3n^2}{2mn^2}}$ **$4|m|$**

21. $\sqrt{\dfrac{16x^{18}}{3600y^{20}}}$ **$\dfrac{|x^9|}{15y^{10}}$**

22. $\sqrt{\dfrac{256x^{40}}{25}}$ **$\dfrac{16x^{20}}{5}$**

23. $\sqrt{2.25x^4}$ **$1.5x^2$**

24. $-\sqrt{2.56k^2}$ **$-1.6|k|$**

11–5 Square Roots of Variable Expressions (continued)

Example 2   Simplify $\sqrt{m^2 - 8m + 16}$.

Solution $\sqrt{m^2 - 8m + 16} = \sqrt{(m - 4)^2} = |m - 4|$

Simplify.

25. $\sqrt{x^2 + 4x + 4}$ **$|x + 2|$**

26. $\sqrt{n^2 - 14n + 49}$ **$|n - 7|$**

27. $\sqrt{x^2 - 6x + 9}$ **$|x - 3|$**

28. $\sqrt{m^2 - 10m + 25}$ **$|m - 5|$**

Example 3   Solve $4x^2 = 25$.

Solution 1
$$4x^2 = 25$$
$$4x^2 - 25 = 0$$
$$(2x + 5)(2x - 5) = 0$$
$$2x = -5 \quad \text{or} \quad 2x = 5$$
$$x = -\frac{5}{2} \quad \text{or} \quad x = \frac{5}{2}$$

Check: $4\left(\dfrac{5}{2}\right)^2 \overset{?}{=} 25$ and $4\left(-\dfrac{5}{2}\right)^2 \overset{?}{=} 25$

$25 = 25 \checkmark$ and $25 = 25 \checkmark$

The solution set is $\left\{\dfrac{5}{2}, -\dfrac{5}{2}\right\}$.

Solution 2
$$4x^2 = 25$$
$$x^2 = \frac{25}{4}$$
$$x = \pm\sqrt{\frac{25}{4}}$$
$$x = \pm\frac{5}{2}$$

Solve.

29. $x^2 = 16$ **$\{-4, 4\}$**

30. $n^2 = 36$ **$\{-6, 6\}$**

31. $x^2 - 9 = 0$ **$\{-3, 3\}$**

32. $d^2 - 25 = 0$ **$\{-5, 5\}$**

33. $0 = a^2 - 49$ **$\{-7, 7\}$**

34. $0 = m^2 - 64$ **$\{-8, 8\}$**

35. $2m^2 - 18 = 0$ **$\{-3, 3\}$**

36. $40b^2 - 160 = 0$ **$\{-2, 2\}$**

37. $36y^2 - 16 = 0$ **$\left\{-\dfrac{2}{3}, \dfrac{2}{3}\right\}$**

38. $4c^2 - 25 = 0$ **$\left\{-\dfrac{5}{2}, \dfrac{5}{2}\right\}$**

39. $0 = 49z^2 - 9$ **$\left\{-\dfrac{3}{7}, \dfrac{3}{7}\right\}$**

40. $0 = 45x^2 - 125$ **$\left\{-\dfrac{5}{3}, \dfrac{5}{3}\right\}$**

Mixed Review Exercises

Simplify.

1. $\pm\sqrt{80}$ **$\pm 4\sqrt{5}$**

2. $-4\sqrt{75}$ **$-20\sqrt{3}$**

3. $3\sqrt{256}$ **48**

4. $2^{-3} - 3^{-2}$ **$\dfrac{1}{72}$**

5. $4^3 \cdot 2^{-5}$ **2**

6. $(3x^2)^3(-2x^4)^2$ **$108x^{14}$**

Evaluate if $x = 9$, $y = 16$, and $n = 1$.

7. $x^2 + y^2$ **337**

8. x^2n^2 **81**

9. $y^2 - x^2$ **175**

10. $\sqrt{\dfrac{y}{n}}$ **4**

11. $\sqrt{\dfrac{x}{y}}$ **$\dfrac{3}{4}$**

12. $\left(\sqrt{y}\right)^2$ **16**